Bikali College, Dhupdhara

Department of Mathematics

Home Assignment- June, 2020

Class:- BA/ B.Sc. 4th Semester

Sub:- Mathematics (General)

Paper:- E-403 **Full Marks:- 50**

1. (a) Define the derivative of a vector function $\vec{f}(t)$ with respect to the scalar variable t 1

(b) If $\vec{r} = [\vec{f}\vec{g}\vec{h}]$, Show that

$\frac{d\vec{r}}{dt} = [\frac{d\vec{f}}{dt}\vec{g}\vec{h}] + [\vec{f}\frac{d\vec{g}}{dt}\vec{h}] + [\vec{f}\vec{g}\frac{d\vec{h}}{dt}]$ 3

2.(a) Show that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\vec{b}\vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$ 3

(b) The volume of the parallelepiped whose edges are represented by $(3i + 2j - 4k)$, $(3j + i + mk)$ and $(i - 2j + k)$ is 49 cubic units, Find the value of m 3

(c) Given the vectors

$\vec{p} = \frac{\vec{b} \times \vec{c}}{\vec{a} . (\vec{b} \times \vec{c})}$, $\vec{q} = \frac{\vec{c} \times \vec{a}}{\vec{a} . (\vec{b} \times \vec{c})}$, and $\vec{r} = \frac{\vec{a} \times \vec{b}}{\vec{a} . (\vec{b} \times \vec{c})}$ 4

Show that if $\vec{a} . (\vec{b} \times \vec{c}) \neq 0$, then

(i) if $\vec{a} . (\vec{b} \times \vec{c}) = V$, then $\vec{p} . (\vec{q} \times \vec{r}) = \frac{1}{V}$

(ii) $\vec{p}$, $\vec{q}$ *and* $\vec{r}$ are non-coplanar if $\vec{a}$, $\vec{b}$ *and* $\vec{c}$ are non-coplanar.

3. (a) The position vector of a particle at time t is given by 3

$\vec{r} = \cos \omega t i + \sin \omega t j$, where ω is a constant. Show that its velocity $\vec{v}$ is Perpendicular to $\vec{r}$.

(b) If $A = 3xyz^2i + 2xy^3j - x^2yzk$ and $\emptyset = 3x^2 - yz$, then find

(i) $\vec{A}.\vec{\nabla}\emptyset$, (ii) $\vec{\nabla}.(\vec{\nabla} \times \vec{A})$ $2\times 2 = 4$

4. (a) Define cone. Show that the general equation of the cone of second degree which passes through the axes is $fyz + gzx + hxy = 0$ 1+3=4

(b) Find the Equation of the right circular cone whose vertex is (3, 2, 1), axis is the line $\frac{x-3}{4} = \frac{y-2}{1} = \frac{z-1}{3}$ and the semi-vertical angle is 30° 5

(c) Find the equation of the cylinder generated by the lines parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$, the guiding curve being the cone 5

$3x^2 + 4xy + 5y^2 = 1, \quad z = 2$

5. (a) Find the distance from the point (3, 4, 5) to the point where the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ meets the plane $x + y + z = 17$ 5

(b) Prove that the shortest distance between the lines $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$ $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$ is $4\sqrt{3}$ 5

(c) Find the equation of the cone whose vertex is (α, β, γ) and guiding curve is $z = 0, \quad ax^2 + by^2 = 1$ 5

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